Smooth and discrete extremal surfaces

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Introduction

A central example of an extremal surface is a minimal surface. These are surfaces with mean curvature equal to zero at all points (this is the sense in which they are extremal: an extreme condition holds everywhere). One can easily produce minimal surfaces by dipping a curved wire into a soap solution. The soap film produced is a minimal surface.

Other examples of extremal surfaces are constant mean curvature surfaces (mean curvature has a constant on the surface) and constant Gaussian curvature surfaces (as perhaps expected, Gaussian curvature is constant on the surface).

The theory of smooth extremal surfaces – those defined by differentiable functions – is a very well studied subject, the discrete version less so. By discrete we mean that we have a finite sample of points from our surface. The concept is similar to a digital picture: if we zoom in we see it is made up of pixels and what we saw as a smooth curve is in fact blocky and jagged.

By taking a discrete sample of points from a surface we can develop a theory similar to that of smooth surfaces in MATH2051 *Geometry of Curves and Surface*. In this project we develop the smooth theory of extremal surfaces and investigate how this theory can guide the discrete theory.

Since one aim of the project is to produce interesting pictures of extremal surfaces knowledge of a programming language or computer algebra package will be essential. Some great pictures of minimal surfaces such as that below can be found at: [http://bugman123.com/MinimalSurfaces/index.html](http://bugman123.com/MinimalSurfaces/index.html)

Prerequisites

- MATH2051 *Geometry of Curves and Surfaces*.
- Experience of a programming language or algebra package such Maple, Python, or Matlab. Having passed MATH1920 *Computational Mathematics* should be sufficient for example.

References
